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ABSTRACT

Non-Linear Dynamics in Deviations from the Law of One Price: A Broad-Based Empirical Study*

In this Paper we test empirically the validity of the law of one price using data for five major bilateral US dollar exchange rates and nine goods sectors during the recent floating exchange rate regime since the early 1970s. Using threshold autoregressive models, we find strong evidence of non-linear mean reversion in deviations from the law of one price with plausible convergence speeds. Consistent with theoretical arguments on international goods markets arbitrage under transactions costs and with an emerging strand of empirical literature, these results contribute towards forming a consensus view in favour of discrete regime switching in deviations from the law of one price and the presence of differing non-zero transactions costs across a broad range of goods and countries.

JEL Classification: F31
Keywords: law of one price, mean reversion, purchasing power parity, real exchange rate and threshold non-linearity

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NON-TECHNICAL SUMMARY

The law of one price (LOOP) states that identical goods, once their national prices are expressed in a common currency, should sell for the same price across different international locations. In reality, however, we observe that numerous and even fairly homogeneous goods sell at different prices in different locations, which is inconsistent with the arbitrage considerations that provide the rationale of the LOOP.

One obvious reason why prices of equivalent commodities may not be the same across different geographical locations is the existence of transactions and transportation costs and other impediments to trade such as tariffs and quotas, which drive a wedge between prices in different locations. Several theoretical contributions emphasise the significance of transactions or international trade costs in modelling deviations from the LOOP. These studies suggest that, as a consequence of the presence of non-zero transactions costs, deviations from the LOOP should contain significant nonlinearities, the key idea being that deviations from the LOOP will be non-mean-reverting as long as they are smaller than the arbitrage costs, and mean reverting once they exceed the arbitrage costs.

Following these theoretical arguments, several empirical studies have investigated the existence of non-linearities in deviations from the LOOP (see Michael, Nobay and Peel, 1994; Obstfeld and AM Taylor, 1997; AM Taylor, 2001; O'Connell and Wei, 2002). In these studies, the non-linear nature of the adjustment process is generally investigated in terms of a 'threshold autoregressive' (TAR) model (Tong, 1990). The TAR model allows for a transactions costs band within which no adjustment in deviations from the LOOP takes place while outside of the band, as goods arbitrage becomes profitable, the process switches abruptly to revert towards the band.

Although these studies are cumulating evidence in favour of non-linearity in deviations from the LOOP, they are generally based on few commodities or currencies, so that the validity of the LOOP and the presence of statistically significant non-linearities in deviations from the LOOP remain contentious. Also, most studies employing TAR models in this context were written before the publication of Hansen's (1996, 1997) work, which provides important econometric contributions for testing linearity against TAR-type non-linearity and for estimating TAR models. Hence, the objective of the present study, which sets this Paper apart in the relevant literature, is to provide a broad-based study on the presence of TAR-type non-linearities in deviations from the LOOP and on the ability of the TAR model to characterize them, using the latest available econometric technology in this context. In an attempt to contribute towards making TAR non-linear dynamics in deviations from the LOOP a stylized fact, we investigate the behaviour of real sectoral dollar exchange rates (deviations from the LOOP) *vis-a-vis* five major currencies for nine sectors over the recent floating period since 1974.

The empirical results are encouraging in that they provide evidence suggesting that, in general, the TAR model characterizes well deviations from the LOOP across this broad range of currencies and sectors. Estimated transactions costs appear to fluctuate widely across sectors and countries. It appears that goods markets between the US and Japan have lower transactions costs than between the US and Europe. In general, adjustment towards the LOOP is observed to be fairly fast although the estimated delay parameter, which measures the timing of the reaction of market participants to deviations from the law of one price, is estimated to be longer than one might perhaps expect. Also, these results suggest that deviations from the LOOP may be somewhat sticky (given the delay parameter is on average larger than four quarters), but they are not as persistent as a large literature has hitherto suggested.

Overall, this study provides strong evidence indicating the validity of the LOOP with allowance for the effects of transactions costs. In particular, our findings accord strongly with the emerging theoretical literature on non-linear real exchange rate adjustment in the presence of international arbitrage costs and contribute towards forming a consensus view that discrete regime switching characterizes deviations from the LOOP across a broad range of goods for a number of industrialized countries over the recent floating period.

1 Introduction

The law of one price (LOOP) states that identical goods, once their national prices are expressed in a common currency, should sell for the same price across different international locations. In reality, however, we observe that numerous and even fairly homogeneous goods sell at different prices in different locations, which is inconsistent with the arbitrage considerations that provide the rationale of the LOOP.

One obvious reason why prices of equivalent commodities may not be the same across different geographical locations is the existence of transactions and transportation costs and other impediments to trade such as tariffs and quotas, which drive a wedge between prices in different locations. Several theoretical contributions - discussed in the next section - emphasize the significance of transactions or international trade costs in modelling deviations from the LOOP. These studies suggest that, as a consequence of the presence of nonzero transactions costs, deviations from the LOOP should contain significant nonlinearities, the key idea being that deviations from the LOOP will be non-mean-reverting as long as they are smaller than the arbitrage costs, and mean reverting once they exceed the arbitrage costs.[1]

Following these theoretical arguments, several empirical studies have employed threshold-type processes to model nonlinearities in deviations from the LOOP (see, *inter alia*, Michael, Nobay and Peel, 1994; Obstfeld and A.M. Taylor, 1997; A.M. Taylor, 2001; O'Connell and Wei, 2002). In these studies, the nonlinear nature of the adjustment process is generally investigated in terms of a threshold autoregressive (TAR) model of some sort (see Tong, 1990). The TAR model allows for a transactions costs band within which no adjustment in deviations from the LOOP takes place - so that deviations may exhibit unit root behavior - while outside of the band, as goods arbitrage becomes profitable, the process switches abruptly to become stationary autoregressive.

Although these studies are cumulating evidence in favor of threshold-type nonlinearity in deviations from the LOOP, they are generally based on few commodities or currencies, so that the

[1] In this paper we refer to international trade costs or transactions costs or arbitrage costs in the broadest possible sense as *all* costs of international trade, including, *inter alia*, transportation costs, tariffs and nontariff barriers and any other costs that agents or firms incur in transactions in international goods markets (see Obstfeld and Rogoff, 2000).

validity of the LOOP and the presence of statistically significant nonlinearities in deviations from the LOOP remain contentious.[2] Also, most studies employing TAR models in this context were written before the publication of Hansen's (1996, 1997) work, which provides important econometric contributions for testing linearity against threshold-type nonlinearity and for estimating TAR models. Hence, the objective of the present study, which sets this paper apart in the relevant literature, is to provide a broad-based study on the presence of threshold-type nonlinearities in deviations from the LOOP and on the ability of the TAR model to characterize them, using the latest available econometric technology in this context. In an attempt to contribute towards making TAR nonlinear dynamics in deviations from the LOOP a stylized fact, we investigate the mean-reverting properties for real sectoral dollar exchange rates (deviations from the LOOP) *vis-a-vis* five major currencies for nine sectors over the recent floating period since 1974. Our results provide evidence suggesting that, in general, the TAR model characterizes well deviations from the LOOP across this broad range of currencies and sectors, yielding plausible estimates of transactions costs and convergence speeds. In turn, these results are consistent with the theoretical literature on transactions costs in international goods markets arbitrage and with the emerging empirical literature stressing the importance of nonlinearities in deviations from the LOOP.

The remainder of this study is set out as follows. Section 2 provides an overview of the relevant literature, including the emerging theoretical and empirical literature on nonlinear dynamics in the LOOP as a motivation for our research. In Section 3, we discuss the TAR model and the linearity testing and model estimation procedure employed. Section 4 describes the data set, while in Section 5 we report the results from the empirical analysis, including linearity tests applied to deviations from the LOOP, the TAR estimation results, a Monte Carlo exercise designed to investigate the power properties of the linearity tests employed here, and some robustness checks. A final section briefly summarizes and concludes.

[2] To gauge the degree of skepticism of the international macroeconomics literature on the validity of the LOOP, see, for example, the vast strand of the 'new open macroeconomics' literature assuming that there are permanent deviations from the LOOP (e.g. Betts and Devereux, 2000; Bergin and Feenstra, 2001; Sarno, 2001).

2 Motivating Nonlinear Dynamics in Deviations from the Law of One Price

The LOOP states that identical goods should sell at the same price, expressed in a common currency, across locations. Thus, deviations from the LOOP for good i, $q_t(i)$ say, may be defined as

$$q_t(i) = s_t - p_t(i) + p_t^*(i), \tag{1}$$

where s_t is the logarithm of the nominal exchange rate (domestic price of foreign currency), and $p_t(i)$ and $p_t^*(i)$ are the logarithms of the domestic-currency price and foreign-currency price of good i at time t respectively.

The rationale behind the LOOP is a simple arbitrage hypothesis: if two identical goods are traded at different prices in different locations, then a profitable arbitrage opportunity arises in that arbitrageurs can buy the good cheaply in one location and sell it at a higher price in the other. In turn, in the absence of arbitrage costs, this process leads to convergence of the deviations from the LOOP towards zero, where exchange-rate-adjusted prices are equalized across countries and no profitable arbitrage opportunities are left.[3]

In general, econometric studies suggest rejection of the LOOP for a very broad range of goods and provide strong empirical evidence both that deviations from the LOOP are highly volatile and that the volatility of relative prices is considerably lower than the volatility of nominal exchange rates - see, for example, Isard (1977), Richardson (1978), Giovannini (1988), Knetter (1989, 1993).

Some studies have tested the LOOP from a different perspective, focusing on intra-national price

[3] The LOOP is essentially the basic building block of purchasing power parity (PPP). In particular, recall that the real exchange rate is defined in logarithmic terms as $q_t = s_t - p_t + p_t^*$, where p_t and p_t^* are the logarithms of the domestic and foreign price levels respectively. This definition clearly indicates that the real exchange rate may be seen as a measure of the deviation from PPP. In practice, the empirical literature studying the behavior of deviations from PPP calculates the real exchange rate according to the above definition using aggregate national price indices. Also, PPP may be given an arbitrage interpretation identical to the one described above for the LOOP, except that PPP applies to baskets of goods across countries, whereas the LOOP applies to individual goods (see Froot and Rogoff, 1995; Rogoff, 1996; Cheung and Lai, 1993a,b; 2000a,b, 2001; Edison, Gagnon and Melick, 1997; Lothian, 1997; Lothian and Taylor, 1996, 2000; Coakley and Fuertes, 1997; A.M. Taylor, 2000; M.P. Taylor and Sarno, 1998; Sarno and M.P. Taylor, 2002).

differentials rather than international exchange-rate-adjusted price differentials. For example, Parsley and Wei (1996) look for convergence towards the LOOP in the absence of trade barriers or nominal exchange rate fluctuations by analyzing a panel of 51 prices from 48 cities in the US. They find convergence rates substantially higher than typically found in cross-country data, that convergence occurs faster for larger price differences, and that rates of convergence are slower for cities further apart. Engel (1993) uses disaggregated data for the US and Canada in order to make a comparison between the volatility of the deviations from the LOOP at international level with the volatility of the deviations from the LOOP between cities of the same country. The most striking result of Engel's study comes from the comparison between the conditional variances for price differentials of similar goods across borders and the conditional variances for price differentials of dissimilar goods intranationally. The latter volatility is estimated to be significantly lower than the former. Extending this line of research, Engel and Rogers (1996) use data for fourteen categories of consumer price indices for both US and Canadian cities in order to analyze the basic stochastic properties of deviations from the LOOP. Engel and Rogers find evidence that the distance between cities can explain a considerable amount of the price differential of similar goods in different cities of the same country. Nevertheless, the price differentials are considerably larger for two cities across different countries relative to two equidistant cities in the same country. The estimates of Engel and Rogers suggest that crossing the national border - the so-called "border effect" - increases the volatility of price differentials by the same order of magnitude which would be generated by the addition of between 2,500 and 23,000 extra miles between the cities considered.

Among the possible explanations of the violation of the LOOP suggested by the literature, transportation costs, tariffs and nontariff barriers play a dominant role. An estimate of the wedge driven by the costs of transportation is given, for example, by the International Monetary Fund (IMF, 1994): the difference between the value of world exports computed as "free on board" (FOB) and the value of world imports charged in full, or cost, insurance and freight (CIF) is estimated at about ten percent and is found to be highly variable across countries.[4] Moreover, even if tariffs have been considerably reduced over time across major industrialized countries, nontariff barriers

[4] Moreover, the presence of significant nontraded components in the prices used by the empirical literature may induce violation of the LOOP (e.g. the cost of labour employed locally).

are still very significant. Governments of many countries often intervene in trade across borders using nontariff barriers in a way that they do not use within their borders (for example in the form of strict inspection requirements) - see Knetter (1994), Feenstra (1995), Rogoff (1996), Feenstra and Kendall (1997), Obstfeld and Rogoff (2000).[5]

Frictions in international arbitrage have important implications and, in particular, imply potential nonlinearities in the deviations from the LOOP. The idea that there may be nonlinearities in goods arbitrage dates at least from Heckscher (1916), who suggested that there may be significant deviations from the LOOP due to international transactions costs between spatially separated markets. A similar viewpoint can be discerned in the writings of Cassel (e.g. Cassel, 1922) and, to a greater or lesser extent, in other earlier writers (Samuelson, 1954; Officer, 1982). More recently, a number of authors have developed theoretical models of nonlinear real exchange rate adjustment arising from transactions costs in international arbitrage (e.g. Benninga and Protopapadakis, 1988; Dumas, 1992; Sercu, Uppal and Van Hulle, 1995; Ohanian and Stockman, 1997; O'Connell, 1998; Obstfeld and Rogoff, 2000). In most of these models, proportional or 'iceberg' transport costs ('iceberg' because a fraction of goods are presumed to 'melt' when shipped) create a band for the real exchange rate within which the marginal cost of arbitrage exceeds the marginal benefit. Assuming instantaneous goods arbitrage at the edges of the band then typically implies that the thresholds become reflecting barriers.

Drawing on recent work on the theory of investment under uncertainty, some of these studies show that the thresholds should be interpreted more broadly than as simply reflecting shipping

[5] The literature has proposed several other explanations of the rejection of the LOOP. One explanation comes from the "pricing to market" (PTM) theory (Krugman, 1987). Following the developments of theories of imperfect competition and trade, the main feature of this theory is that the same good can be given a different price in different countries when oligopolistic firms are supplying it. See also the empirical study by Cheung, Chinn and Fujii (2001), who examine the relationship between market structure and the persistence of US dollar-denominated sectoral real exchange rates using annual data for 14 major industrialized countries. Cheung, Chinn and Fujii find that differences in market structure significantly determine the rates at which deviations from the LOOP decay, implying that an imperfectly competitive market structure may explain the observed persistence in sectoral real exchange rates. Alternatively, it seems possible that the failure of the LOOP may be explained by institutional factors typical of this century which have increased the persistence of deviations from the LOOP. However, Froot, Kim and Rogoff (1995), using data on prices for grains and other dairy goods in England and Holland for a span of data which goes from the fourteenth century to the twentieth century, provide empirical evidence suggesting that the volatility of the LOOP is quite stable during the whole period, regardless of the many regime shifts during the sample.

costs and trade barriers *per se*, but also as resulting from the sunk costs of international arbitrage and the resulting tendency for traders to wait for sufficiently large arbitrage opportunities to open up before entering the market (see in particular Dumas, 1992; see also Dixit, 1989, and Krugman, 1989). O'Connell and Wei (2002) extend the iceberg model to allow for fixed as well as proportional costs of arbitrage. This results in a two-threshold model where the real exchange rate is reset by arbitrage to an upper or lower inner threshold whenever it hits the corresponding outer threshold. Intuitively, arbitrage will be heavy once it is profitable enough to outweigh the initial fixed cost, but will stop short of returning the LOOP deviations to equilibrium because of the proportional arbitrage costs. Coleman (1995) suggests that the assumption of instantaneous trade should be replaced with the presumption that it takes time to ship goods. In this model, transport costs again create a band of no arbitrage for deviations from the LOOP, but the exchange rate can stray beyond the thresholds. Once beyond the upper or lower threshold, deviations from the LOOP become increasingly mean reverting with the distance from the threshold. Within the transactions costs band, when no trade takes place, the process is divergent so that the exchange rate spends most of the time away from parity.

Some empirical evidence of the effect of transactions costs in this context is provided by Davutyan and Pippenger (1990). More recent studies modelling nonlinearities in the deviations from the LOOP include Michael, Nobay and Peel (1994), Obstfeld and A.M. Taylor (1997), O'Connell (1998), A.M. Taylor (2001) and O'Connell and Wei (2002). In all these studies, the nonlinear nature of the adjustment process is investigated in terms of a TAR model (Tong, 1990). As mentioned above, the TAR model allows for a transactions costs band within which no adjustment in deviations from the LOOP takes place - so that deviations may exhibit unit root behavior - while outside of the band, as goods arbitrage becomes profitable, the process switches abruptly to become stationary autoregressive. Michael, Nobay and Peel (1994) investigate the relative US dollar price of six wheat varieties and provide evidence for nonlinear mean reversion in deviations from the LOOP. Obstfeld and A.M. Taylor (1997) employ disaggregated (clothing, food and fuel) as well as aggregate data for 32 locations at a monthly frequency from 1980 to 1995, the US being the reference country. Their estimated thresholds are found to be between 7 and 10 percent on average across different types of goods. Obstfeld and A.M. Taylor (1997) discover transactions costs to be lower between the US and Asia (about 2 to 8 percent) than between the US and Europe (about 9 to 19 percent). Using two different sets of relative price panels over the period 1975 to 1992, O'Connell and Wei (2002) also find strong evidence of nonlinear mean reversion in deviations from the LOOP.

Recently, A.M. Taylor (2001) has shown, on the basis of Monte Carlo experiments with a nonlinear artificial data generating process, that there can also be substantial upward bias in the estimated half life of adjustment from assuming linear adjustment when in fact the true adjustment process is nonlinear.[6] [7]

Overall, these models suggest that deviations from the LOOP become mean reverting only when they are large enough to generate profitable arbitrage opportunities net of transactions costs. These models therefore provide a rationale for threshold-type models of deviations from the LOOP that allow for a jump from the non-mean-reverting behavior within the transactions costs band to mean-reverting behavior outside the transactions costs band. The literature to date has only begun to explore the issue of nonlinearities in deviations from the LOOP.

3 Modelling Nonlinear Adjustment in the Law of One Price

The theoretical contributions discussed in the previous section clearly motivate the existence of nonlinear mean reversion consistent with threshold-type behavior in deviations from the LOOP.

[6] A.M. Taylor (2001) also investigates the impact of temporal aggregation in the data. Using a model in which deviations from the LOOP or PPP follows an AR(1) process at a higher frequency than that at which the data is sampled, A.M. Taylor shows analytically that the degree of upward bias in the estimated half life rises as the degree of temporal aggregation increases - i.e. as the length of time between observed data points increases. This time aggregation problem is a difficult issue for researchers to deal with since long spans of data are required in order to have a reasonable level of power when nonstationarity tests are applied, and long spans of high-frequency data do not exist. On the other hand, A.M. Taylor also shows that the problem becomes particularly acute when the degree of temporal aggregation exceeds the length of the actual half life, so that this source of bias may be mitigated somewhat if the researcher believes that the true half life is substantially greater than the frequency of observation. See also Prakash and A.M. Taylor (1997).

[7] A parallel literature has investigated nonlinearities in deviations from purchasing power parity. Using annual data for dollar-franc and dollar-sterling spanning two centuries, Michael, Nobay and Peel (1997) and Lothian and M.P. Taylor (2000) provide evidence of nonlinear mean reversion for each rate. M.P. Taylor, Peel and Sarno (2001) focus on four major dollar rates during the post-Bretton Woods period and uncover nonlinear mean reversion in each case; see also Sarno (2000a,b). See also the cross-country study by O'Connell (1998).

In a world with nonzero transactions costs, deviations from the LOOP will be non-mean reverting as long as they are smaller than the arbitrage costs, but they will be mean reverting once they exceed the arbitrage costs.

Define the deviations from the LOOP for a sector i (i.e. the sectoral real exchange rate for sector i) as $q_t(i)$. An econometric model which well captures the predictions of a discrete switch in $q_t(i)$ is the following symmetric three-regime TAR model:[8]

$$\Delta q_t(i) = \begin{cases} \left[(\beta_1 - 1)(q_{t-1}(i) - \theta_i) + \sum_{j=2}^{p} \beta_j (q_{t-j}(i) - \theta_i)\right] l(q_{t-d}(i) > \theta_i) \\ + \left[\sum_{j=1}^{p-1} \alpha_j \Delta q_{t-j}(i)\right] l(|q_{t-d}(i)| \leq \theta_i) \\ + \left[(\beta_1 - 1)(q_{t-1}(i) + \theta_i) + \sum_{j=2}^{p} \beta_j (q_{t-j}(i) + \theta_i)\right] l(q_{t-d}(i) < -\theta_i) + \varepsilon_{it} \end{cases} \tag{2}$$

where Δ denotes the first difference operator; θ_i is the threshold parameter for sector i; $q_{t-d}(i)$ represents the threshold variable for sector i, with d denoting an integer chosen from the set $\Psi \in \left[1, \bar{d}\right]$. The indicator function, given by $l(\cdot)$, takes a value of unity when the bracketed expression is true, and is zero otherwise. The error term ε_{it} is assumed independently and identically distributed *(iid)* Gaussian. The autoregressive order of q_t is $p \geq 1$. Note that essentially the TAR specified in (2) has two regimes given that the behavior of the outer regimes is identical because of the symmetry assumption.

A TAR model of the form (2) in which the threshold variable is assumed to be the lagged dependent variable is termed a Self-Exciting TAR (SETAR) model. The integer d represents the delay parameter and reflects the possibility that market participants react to deviations of the LOOP from equilibrium with a lag. As long as $|q_{t-d}| \leq \theta$, the time series q_t follows a unit root process. Thus, in this regime (or region), there is no tendency for the series q_t to move back towards equilibrium. Once $|q_{t-d}| > \theta$, however, q_t becomes a stationary process and has a tendency to revert back as long as $\sum_{j=1}^{p} \beta_j < 1$, regardless of whether q_{t-d} is positive or negative. As arbitrage is not profitable in the region defined within the bands, the series will only revert back towards an equilibrium band, here $[-\theta, \theta]$.[9] Also, note that the TAR model (2) is deliberately

[8] The TAR model was initially introduced by Tong (1983), although a number of econometric issues concerning TAR specification and estimation have only recently been resolved, primarily by Hansen (1996, 1997).

[9] A sufficient condition for global stability (stationarity) is that the roots of the autoregression in the outer regime are less than unity in absolute value (see Tjostheim, 1986; Tong, 1990; Granger and Teräsvirta, 1993).

constructed as a symmetric process. This is the case since it is difficult to think of plausible economic arguments why arbitrage forces should vary depending upon whether deviations from the LOOP are above or below the arbitrage bands.[10]

To illustrate the TAR estimation procedure, it will be convenient to reparameterize (2) as follows

$$\Delta q_t = M_t(\theta, d)'\Phi + \varepsilon_t \tag{3}$$

where $M_t(\theta, d)'$ is a (1×3) row vector containing three $(1 \times p)$ subvectors of variables defining the behavior of q_t in each of the three regimes; and Φ is a (3×1) column vector, where each element is a $(p \times 1)$ subvector consisting of the autoregressive parameters to be estimated.[11] The parameters to be estimated in (3) are the threshold parameter θ, the delay parameter d, and the autoregressive parameters in Φ; the i sector-specific term has been dropped for clarity in (3). As shown in Hansen (1996, 1997), sequential conditional least squares is an attractive way to estimate the parameters since the regression equation is nonlinear and discontinuous. For any given (θ, d) combination, the least squares estimator of Φ is

$$\widehat{\Phi}(\theta, d) = \left(\sum_{t=1}^{N} M_t(\theta, d) M_t(\theta, d)'\right)^{-1} \left(\sum_{t=1}^{N} M_t(\theta, d)\Delta q_t\right) \tag{4}$$

with estimated error $\widehat{\varepsilon}_t(\theta, d) = \Delta q_t - M_t(\theta, d)'\widehat{\Phi}\,(\theta, d)$, and residual sum of squares

[10] Balke and Fomby (1997) define the TAR model (2) as the Band-TAR model. An alternative TAR model is the so-called Equilibrium-TAR (EQTAR) model. In the EQTAR model the time series is allowed to tend towards a precise equilibrium value when the process is outside the region $[-\theta, \theta]$. However, as deviations from the LOOP do not *fully* die out in the presence of arbitrage costs, a Band-TAR model is more appealing than an EQTAR model in the present context.

[11] Precisely, $M_t(\theta, d)' = \begin{bmatrix} X'l(q_{t-d} > \theta) & Y'l(|q_{t-d}| \leq \theta) & Z'l(q_{t-d} < -\theta) \end{bmatrix}$, where $X' = \begin{bmatrix} (q_{t-1} - \theta) & (q_{t-2} - \theta) & \ldots & (q_{t-p} - \theta) \end{bmatrix}$, $Y' = \begin{bmatrix} \Delta q_{t-1} & \Delta q_{t-2} & \ldots & \Delta q_{t-p} \end{bmatrix}$, and $Z' = \begin{bmatrix} (q_{t-1} + \theta) & (q_{t-2} + \theta) & \ldots & (q_{t-p} + \theta) \end{bmatrix}$. Also, $\Phi' = \begin{bmatrix} \beta & \alpha & \beta \end{bmatrix}$, where $\beta' = \begin{bmatrix} \beta_1 - 1 & \beta_2 & \beta_3 & \ldots & \beta_p \end{bmatrix}$, and $\alpha' = \begin{bmatrix} \alpha_1 & \alpha_2 & \ldots & \alpha_{p-1} & 0 \end{bmatrix}$. Therefore we are effectively restricting the deviations from the LOOP to have a unit root within the band. A more general process might produce estimates of the degree of persistence within the band to verify how close to a unit root the sectoral real exchange rate is within the band. However, in this study we choose to use the restricted version of the TAR which implicitly assumes that unit root behavior approximates sufficiently well the sectoral real exchange rate behavior within the band, consistent with the theoretical studies cited in the previous subsection.

$$\widehat{\sigma}^2(\theta, d) = \frac{1}{N}\sum_{t=1}^{N}\left(\Delta q_t - M_t(\theta, d)'\widehat{\Phi}(\theta, d)\right)^2 = \frac{1}{N}\sum_{t=1}^{N}\widehat{\varepsilon}_t(\theta, d)^2. \tag{5}$$

The threshold parameter and delay parameter are estimated simultaneously. Hansen (1997) suggests carrying out a two-dimensional search over (θ, d) and estimating the TAR model for any given (θ, d) combination by applying the method of sequential conditional least squares, choosing the threshold parameter and delay parameter that minimize the residual variance:

$$(\widehat{\theta}, \widehat{d}) = \underset{\theta \in \Gamma, d \in \Psi}{\arg\min}\, \widehat{\sigma}^2(\theta, d), \tag{6}$$

where $\Gamma = \left[\underline{\theta}, \bar{\theta}\right]$ and $\Psi = \left[1, \bar{d}\right]$.

Once the grid search over (θ, d) has been completed and the optimal threshold parameter $\widehat{\theta}$ and the optimal delay parameter $\widehat{d}$ have been found, the least squares estimator of Φ is found to be $\widehat{\Phi} = \widehat{\Phi}(\widehat{\theta}, \widehat{d})$ with sum of squared errors $\widehat{\sigma}^2 = \widehat{\sigma}^2(\widehat{\theta}, \widehat{d})$. Note that, as Ψ is discrete, the estimator of $\widehat{d}$ is superconsistent (Hansen, 1997).

It should be clear that the above TAR model has a linear model nested in its nonlinear structure. Before the nonlinear model is actually estimated, it is wise to find out whether the TAR-type nonlinearity is statistically significant when tested against a linear specification, however. Various linearity tests have been suggested in the literature in this context, which can be subdivided into two broad classes. The first category of linearity tests includes tests where the true data generating process under the alternative hypothesis is of a general nonlinear nature but the exact form of the nonlinearity is left unspecified.[12] The second category of linearity tests contains tests that explicitly specify a TAR model under the alternative hypothesis. This category includes the linearity tests due to Tsay (1989) and Hansen (1996, 1997). A major problem with most conventional linearity tests (including the one due to Tsay) is caused by the existence of nuisance parameters (such as the threshold parameter) under the alternative hypothesis, which essentially means that these parameters are not identified under the null hypothesis. Davies (1977, 1987) and Andrews

[12] Linearity tests that belong to this group include, *inter alia*, the RESET test due to Ramsey (1969) and the Brock-Dechert-Scheinkman (1991) test. (Although the RESET test was designed to be used to test for general model misspecification, in practice it is often employed to detect nonlinearities.)

and Ploberger (1994), among others, have shown that the asymptotic distributions of standard tests of the null of a linear autoregressive process against TAR alternatives are nonstandard.

Hansen (1997) suggests a testing procedure that takes into account the problems resulting from the presence of unidentified nuisance parameters under the null hypothesis of linearity. The Hansen test is based on a standard test statistic that has a nonstandard asymptotic distribution. The test requires pre-estimation of both the linear model under the null hypothesis as well as the TAR model under the alternative. In the context of the TAR model investigated here, under the null hypothesis, the TAR model reduces to a linear nonstationary AR(p) model. Following Hansen (1997), the null of a linear autoregressive model against the alternative of a TAR model can be tested using the statistic:

$$H_N(\theta, d) = N\left(\frac{\widehat{\sigma}^2 - \widehat{\sigma}^2(\theta, d)}{\widehat{\sigma}^2(\theta, d)}\right) \tag{7}$$

with

$$H_N = \sup_{\theta\in\Gamma,\ d\in\Psi} H_N(\theta, d) \ \Rightarrow\ \underset{\theta\in\Gamma, d\in\Psi}{\arg\min}\,\widehat{\sigma}^2(\theta, d) = (\widehat{\theta}, \widehat{d}) \tag{8}$$

where $\widehat{\sigma}^2$ is the residual variance of the linear model. However, since θ and d are not identified under the null, H_N does not have the standard χ^2 asymptotic distribution even if it is essentially constructed as an F-statistic.[13] As standard critical values cannot be computed in this case (as they depend on the particular null and alternative models and on nuisance parameters, *inter alia*), Hansen (1996) derives the distribution of H_N using parametric bootstrap techniques, hence obtaining the distribution of, say, H_N^*. The distribution of H_N^* converges weakly in probability to the null distribution of H_N. The bootstrap approximation to the p-value of the test is calculated by counting the percentage of bootstrap samples for which H_N^* exceeds the observed H_N.

[13] We have not followed Hansen's (1997) notation of denoting this statistic F_N since we wish to avoid any possible confusion with the standard F-statistic. We also prefer to use H_N since it acknowledges Hansen's contribution.

4 Data and Preliminary Unit Root Tests

The primary data sources we used to construct our data set are the Organization for Economic Cooperation and Development's (OECD) *International Sectoral Database* (ISDB) and the International Monetary Fund's (IMF) *International Financial Statistics* (IFS) data base. The countries analyzed include the UK, France, Germany, Italy, Japan, and the US as the reference country. For each country, we obtained the value added at current prices and the value added at constant prices (1990 prices) for the following nine sectors from the OECD's ISDB: basis metal industries (BMI), chemicals and chemical petroleum, coal, rubber and plastic products (CHE), food, beverages and tobacco (FOD), manufacturing goods (MAN), electrical goods (MEL), fabricated metal products, machinery and equipment (MEQ), paper and paper products, printing and publishing (PAP), textile, wearing apparel and leather industries (TEX), and wood and wood products, including furniture (WOD). Price deflators were then calculated as the difference between the logarithm of value added at current prices and the logarithm of value added at 1990 prices. Data on Japan for the sectors MEL and WOD were not available and, therefore, the empirical analysis for Japan is restricted to the remaining seven sectors. Quarterly data on nominal bilateral US dollar (end-of-period) exchange rates *vis-a-vis* the UK pound, French franc, German mark, Italian lira, and Japanese yen were collected from the IMF's IFS.

The time series of interest are the quarterly sectoral real exchange rates (deviations from the LOOP) $q_t(i)$ constructed in logarithmic terms according to equation (1). Deviations from the LOOP (sectoral real exchange rates) were normalized to unity at the beginning of the sample period, which spans from 1974QI to 1993QIV (due to data availability). The price data provided by the OECD's ISDB are of very high quality but are only available at annual frequency. Thus, these data were interpolated into quarterly frequency, using a linear interpolation method. Whether interpolation actually reduces nonlinearity remains unclear and is currently under investigation by various econometricians (see, for example, Granger and Lee, 1999). It is often argued that the information set of the interpolated data is identical to the information set of the original data. Therefore, interpolation does not necessarily affect the degree of nonlinearity. One may then legitimately ask why we interpolate the data if the information in the data remains unchanged.

One benefit of interpolation may be the reduction of small-sample bias. Also, it seems that much of the economics profession is averse to empirical studies with too few observations.[14] [15]

As a preliminary exercise, we tested the hypothesis that deviations from the LOOP follow a nonstationary process by applying several conventional unit root tests to the sectoral real exchange rates (not reported to conserve space). The unit root tests generally suggested the presence of a unit root in the levels of the sectoral real exchange rates considered. First and second differences of the sectoral real exchange rates were found to be stationary. It is well known that the power of standard unit root tests is fairly poor in the present context, however, especially if the true data generating process is in fact nonlinearly mean reverting (see Pippenger and Goering, 1993; A.M. Taylor, 2001; M.P. Taylor, Peel and Sarno, 2001, and the references therein).

5 Nonlinear Estimation Results

5.1 Linearity Tests

Hansen (1997) suggests a test statistic of the null hypothesis of a linear autoregressive model against the alternative of a TAR model. As discussed above, since the asymptotic distribution of the Hansen H-statistic is nonstandard, a parametric bootstrap technique is needed to determine the empirical p-value. In the parametric bootstrap procedure $1,000$ samples of $100+T$ random numbers assumed to be *iid* Gaussian and distributed as $N(0,1)$ are generated, and the first 100 data points are discarded to reduce the dependence on the initialization.[16] For each of the $1,000$ replications, the H-statistic is generated using the calculated residual variance of the linear model under the null and the variance of the TAR alternative model. The p-value of the test is calculated by counting the percentage of bootstrap samples for which the generated H-statistic exceeds the observed H-statistic.

The bootstrap-calculated p-values for the test of the null of linearity against the TAR alternative are reported in the last row of Tables 1-5. The outcome of the linearity tests is very satisfactory in that in 33 out of 43 cases the null hypothesis of linearity is rejected against the TAR alternative at

[14] We are grateful to Clive Granger for a useful conversation on these issues.
[15] See also footnote 24 and Section 5.4
[16] T is defined as the number of observations, which, in the present application, is equal to 80.

the 5 percent significance level. At the 10 percent significance level the rejection rate is around 90 percent, implying that 9 in 10 sectoral real exchange rates series follow a TAR process. In many cases, p-values of less than 1 percent are recorded.[17] Overall, these results suggest that a linear autoregressive model for the sectoral real exchange rates examined is misspecified and rejected against a TAR model.

5.2 The Power of the Linearity Tests: Some Monte Carlo Evidence

In order to assess the robustness of the linearity tests discussed in the previous subsection, we investigated the power properties of the bootstrap procedure suggested by Hansen (1997) by performing a battery of Monte Carlo experiments. The specific aim of the Monte Carlo experiments is to investigate how variations in the autoregressive order p and the outer root of the TAR process affect the power of the Hansen test (1997).[18] All experiments in this section were constructed using $1,000$ replications and applying a finite-sample 5 percent critical value for the test statistics computed using the appropriate percentile of the empirical test calculated under the null hypothesis of linearity. Throughout the analysis, the data generating process is a TAR process calibrated using the estimated TAR model fitted to the real sectoral exchange rate data for the UK paper products sector reported in Table 1 (and discussed more fully in the next section).[19] Errors were generated using a Gaussian random number generator. At each replication $100+T$ random numbers were generated of which the first 100 numbers were always discarded. The generated sample size therefore corresponds to that of the actual data set, with 80 data points.

For any given autoregressive order chosen from the set $p \in \{2, 4, 8\}$, we investigated how variations in the outer root (i.e. the root outside of the band) affect the power of the test to reject the false null hypothesis that the time series is linear autoregressive when in fact the true data generating process follows a TAR process. The outer roots were chosen from the set $r \in$

[17] Note that for Japan, for example, the null hypothesis of a linear $AR(p)$ model is rejected for all sectoral real exchange rates.

[18] The outer root is defined as the sum of the estimated outer autoregressive parameters, $r = \sum_{j=1}^{p} \beta_j < 1$ in the TAR formulation (2). Note that $p = 8$ for all of the TAR models reported in Tables 1-5, as discussed in the next subsection.

[19] We also performed simulations under several other TAR processes, calibrated on other models reported in Tables 1-5. The results were virtually identical, suggesting that the results discussed below should not be subject to a specificity problem (Hendry, 1984).

$\{0.95, 0.75, 0.5, 0.25, 0.05\}$. Generally, one would expect that as the threshold effect, which is defined as the difference in slopes between the two regimes, becomes larger - i.e. the departure from linearity becomes larger - the power of rejecting the false null hypothesis increases. Indeed, the simulation results, presented in Table 6, provide clear evidence that this conjecture is correct for all examined lag lengths. The larger the threshold effect, the higher the power of the linearity test. For example, with four lags, an outer root r of 0.05, which is equivalent to a threshold effect of 0.95 since within the bands the time series follows a unit root process (hence having no tendency to mean revert within the band), results in a rejection rate of about 92.0 percent. A small threshold effect r of say 0.95 (consistent with a finite but very low speed of band reversion even outside the band) still results in a rejection frequency of about 55 percent.

The above experiments were also conducted for different autoregressive orders to investigate how changes in the lag length affect the power of the Hansen test. As it is evident from the results in Table 6, increasing the lag length has some effect on the power of the test, suggesting that there is some cost of overfitting a TAR model - i.e. the power of the test is higher the lower the lag length of the TAR model (indeed this seems to be the case even though the true data generating process in the simulations has $p = 8$).

This simulation evidence provides an important insight into why the linearity test against the TAR alternative could not reject the linear null hypothesis for *each* of the sectoral real exchange rates examined. For threshold effects lying between 0.05 and 0.25, which roughly correspond to the threshold effects generally recorded in the present study, the rejection frequencies range from about 55 percent to about 61 percent when one uses 8 autoregressive lags.[20] That is to say, there would be approximately a 40 to 45 percent probability of failing to reject the hypothesis of linear behavior of sectoral real exchange rates if in fact the sectoral real exchange rates were generated by a nonlinear TAR model of the kind estimated in the present study. Under these circumstances the recorded rejection rate of almost 77% (in 33 out of 43 cases the linear null hypothesis is rejected using a 5 percent empirical critical value) seems very satisfactory. Indeed, it is tempting to argue that, given the results of the Monte Carlo analysis applied to the Hansen test, even those sectoral

[20] Precisely, 81 percent of the sectoral real exchange rates examined here reveal threshold effects that lie between 0.05 and 0.25.

real exchange rates for which we could not reject the hypothesis of linearity at the 5 percent significance level are in fact likely to be well characterized as TAR processes.

Also, notice that our Monte Carlo results are generally consistent with the results of Hansen's (1996) simulation study, although Hansen allows for an asymmetric TAR rather than a symmetric one and his results are based on Monte Carlo experiments only for $p = 1, 2, 3$. In our view, this Monte Carlo evidence demonstrates the usefulness of the Hansen linearity test. The linearity test performs extremely well if applied to a threshold autoregressive process characterized by a small autoregressive order, regardless of the size of the threshold effect. For a threshold autoregressive process with a larger autoregressive order, the power of the linearity test still remains reasonably high for threshold effects of the size observed in the present study.

5.3 TAR Estimation

Tables 1-5 summarize the results of the TAR estimation.[21] In all TAR estimations the lag length p was set equal to 8, which appeared to be the lag length that would guarantee well behaved residuals for all sectoral real exchange rates examined. The set of possible delay parameters we considered was in the range between one and eight, $\Psi = [1, 8]$. The greater the value of the delay parameter, the longer it takes for market participants to react to deviations from the LOOP level. Economic intuition provides a presumption in favor of smaller values of the delay parameter d rather than larger values, since it is not straightforward why there should be long lags before deviations from the LOOP begin to adjust in response to a shock. On the basis of this reasoning, for example, Obstfeld and A.M. Taylor (1997) *a priori* assume the delay parameter to be equal to unity, and A.M. Taylor (2001) derives most of his results assuming a delay parameter of unity.

Depending on the particular country and sector examined, one might typically expect transactions costs to lie between 0 and 20 percent. Therefore, the set of possible threshold parameters was set to $\Gamma = [0.0, 0.2]$. The absence of transactions costs implies that markets are fully integrated. For every single sectoral real exchange rate, the TAR model (2) with $p = 8$, $\Psi = [1, 8]$, and $\Gamma = [0.0, 0.2]$ was estimated, resulting in $21 \times 8 = 168$ function evaluations. The method

[21] As the number of estimates of the autoregressive parameters would amount to more than 700 values, they are not reported here but are available from the authors upon request.

of sequential conditional least squares was applied to estimate the TAR model, as suggested by Hansen (1997).

As is evident from Tables 1-5, transactions costs differ widely across sectors. Consider, for example, Germany, where estimated transactions costs for the PAP sector are found to be of the order of 1%, whereas estimated transactions costs for the FOD sector are some 20%. This implies that the US and German market for paper and paper products, printing and publishing is highly integrated, unlike the market for food, beverages and tobacco. The results also suggest that costs of arbitrage widely fluctuate across countries for a given sector. The PAP sector in the UK reveals transactions costs of 19%, unlike the 1% for Germany. Relatively high costs of trade can be observed for the manufacturing sector and wood and wood product sector, in particular for the European countries, with values for $\widehat{\theta}$ ranging from 17% to 20% and from 12% to 20% respectively, suggesting that US and European goods markets are less integrated for these sectors. It is interesting to note that transactions costs for the textile sector and for the food, beverages and tobacco sectors are relatively high for the European countries (from 14% to 20% for the latter, and from 11% to 20% for the former), while they are fairly low for Japan (from 6% to 7%). Note that among European countries, Italy's transactions costs are relatively high (except for the BMI and CHE sector) and in many cases exceed the costs of Italy's trading partner countries within the European Union.

It is worthwhile noting that the threshold estimates presented in this section are generally consistent with the results of Obstfeld and A.M. Taylor (1997), although Obstfeld and A.M. Taylor estimate a Band-TAR model in which the delay parameter is arbitrarily set to unity and do not apply the Hansen procedure to test formally the hypothesis of linearity. Even though their results are based on different disaggregated data (clothing, food and fuel), one of their key findings is that costs of trade are found to be lower between the US and Asia (2% to 8%) than between the US and Europe (9% to 19%), suggesting that trade barriers between the US and Asia may be smaller than between the US and Europe.[22]

[22] Also, note that plotting of the sectoral real exchange rate data with the estimated transactions costs suggested that actual observations are fairly equally distributed between the inner (nonstationary) regime and the outer (stationary) regime, suggesting that our detection of TAR nonlinearity is not driven by a few outliers.

The TAR estimation results also suggest that the estimated delay parameters vary across sectors and countries and only in 6 out of 43 cases is a delay lag of 1 indicated. Many estimated values for the delay parameter seem to cluster in the range between 4.5 and 6, suggesting that markets react with a lag of 1 to 1.5 half years to deviations from equilibrium. In the textile sector, for example, market participants appear to exploit arbitrage opportunities fairly quickly and deviations from the LOOP dissipate very rapidly, compared to, for example, a two year reaction lag in the electrical goods sector. Note that the estimated delay parameters for the basis metal industry and the electrical goods sector are very similar across countries (about 5 and 8 respectively).

In the estimated TAR model, sectoral real exchange rates are assumed to follow a nonstationary process within the band. Outside the transactions cost band, arbitrage is profitable and the process generating the sectoral real exchange rate is stationary. In this region the series of the sectoral price spread is reverting towards the thresholds. The outer root of the TAR process, r provides, therefore, a measure of the degree of nonlinear reversion towards the thresholds. Calculation of the half life hl ($hl = \ln 0.5/\ln r$) in the outer regime implied by our TAR estimates indicates an average convergence speed of 6 quarters.[23] The smallest outer roots (fastest reversion rates) are for Italian manufacturing ($r = 0.19$), fabricated metal ($r = 0.52$), and wood and wood products ($r = 0.61$) sectors, and for the French food, beverages and tobacco sector ($r = 0.39$). These results contrast with the large empirical literature that suggests low speed of mean reversion of deviations from the LOOP, while confirming the importance of deviating from a linear specification in this context (Rogoff, 1996; Taylor, 2001).[24]

The residual diagnostic statistics are satisfactory except for some slight evidence of ARCH in the Italian BMI and the Japanese CHE sector. The ratio of the residual variance of the estimated TAR model to the residual variance of the alternative linear AR(p) model (V) suggests that, for

[23] Specifically, the average half life is about 4.8 quarters for the UK, 6.0 quarters for Germany, 5.0 quarters for France, 5.7 quarters for Italy, and 10.1 quarters for Japan. Also, the average half life by sector is 9.4 quarters for BMI, 9.2 quarters for CHE, 5.8 quarters for FOD, 5.1 quarters for MEL, 5.4 quarters for MEQ, 2.4 quarters for MAN, 4.7 quarters for PAP, 8.5 quarters for TEX, and 4.0 quarters for WOD.

[24] We also estimated TAR models at annual frequency. The estimation results suggested much longer half lives than the ones recorded for monthly data in Tables 1-5, consistent with the A.M. Taylor's (2001) result that temporal aggregation induces substantial upward bias in the convergence speed in this context. The average convergence speed for the UK (France), for example, was found to be about 2.25 (1.7) years, in contrast with the estimates of 4.8 (4.9) obtained at quarterly frequency. However, as suggested by a simulation exercise carried out by an anonymous referee, it is not obvious that interpolation necessarily generates upward bias in the convergence speed.

all sectoral real exchange rate series examined, the estimated TAR model leads to a significant reduction in the residual variance compared to the corresponding linear model.

5.4 Robustness

Although strong evidence in favor of TAR-type nonlinearity is found in the behavior of the sectoral real exchange rates examined in the previous subsection, there perhaps still remains a concern about the robustness of the inferences drawn in this study, especially in light of the fact that the OECD's *ISDB* annual price data used above have been interpolated into quarterly frequency. This could potentially affect the results concerning the linearity tests and the half-life analysis. Therefore, this subsection aims at assessing the degree to which our inferences are robust to the interpolation of the sectoral price data.

To investigate this issue, we proceed as follows. First, sectoral price data at quarterly frequency are collected and used to generate the sectoral deviations from the LOOP. These data are available for the UK and the US, but not for the other countries examined in our broad-based study. Second, price data at annual frequency are collected from the same source and then converted into quarterly price data using the same linear interpolation technique that was earlier applied to the OECD's *ISDB* annual price data. The interpolated price data are used to construct interpolated sectoral real exchange rates. Finally, the empirical analysis is carried out for the interpolated and non-interpolated sectoral real exchange rates. Comparing the results obtained for the interpolated and non-interpolated sectoral real exchange rates should then shed light on the extent of the problems raised by interpolation.[25]

5.4.1 Data

For two of the countries examined in this study, the UK and the US, we could locate industry-specific producer price (PPI) data at quarterly and annual frequency for the following five sectors: food, drinks and tobacco (RFOD), manufacturing (RMAN), paper (RPAP), textile (RTEX) and

[25] One way to gain further insight into the impact of interpolation on the properties of the sectoral real exchange rate is via Monte Carlo simulations. However, given the availability of non-interpolated data for two countries and given that a study of the effects of interpolation on the stochastic properties of the data is beyond the purpose of this paper, we use only actual data in this subsection. The reader is cautioned that the robustness results reported below may be specific to this particular context and for the range of parameters recorded in this paper.

lumber and wood (RWOD). Price data for the UK were provided by the Office for National Statistics (ONS), while price data for the US were obtained from the Bureau of Labor Statistics (BLS). Note that these price data differ from the OECD's price data. The empirical analysis in this particular subsection is based on industry-specific producer prices, whereas the empirical analysis in the remainder of the paper is based on sectoral deflators. Nevertheless, this subsection is primarily concerned with the impact of interpolation on the statistical properties of the sectoral real exchange rate. Hence, the objective is to compare interpolated and non-interpolated sectoral real exchange rates for a given price deflator.

The interpolated quarterly price data are obtained by interpolating the annual price observations using the same linear frequency conversion technique which was used to interpolate the OECD's *ISDB* annual price data. All price data, interpolated and non-interpolated, cover the time period 1974Q1 to 1993Q4. The base year for the price series has been set to 1990Q1. The nominal exchange rate covers the same time period and is available at quarterly frequency and therefore it is not interpolated.

Quarterly sectoral real exchange rates generated using the non-interpolated price data are labelled *non-interpolated* sectoral real exchange rates or *non-interpolated* sectoral deviations from the LOOP, $q_{NI}(j)$ for some sector j.[26] On the other hand, quarterly sectoral real exchange rates constructed using the interpolated price data according to equation (1) are denoted *interpolated* sectoral real exchange rates or *interpolated* sectoral deviations from the LOOP, $q_I(j)$ for some sector j. All sectoral real exchange rates are expressed in logs.

5.4.2 TAR Estimation Results

Below we present the estimation results obtained from fitting the TAR model (2) to the interpolated and non-interpolated sectoral real exchange rates. In all TAR estimations the lag length p was set equal to 8, as this appears to be the lag length that would guarantee well behaved residuals for all sectoral real exchange rates examined. As in the previous TAR estimation the set of possible delay parameters ranges between one and eight, $\Psi = [1, 8]$, while the set of possible threshold parameters is set to $\Gamma = [0.0, 0.2]$.

[26] For clarity the subscript t denoting time has been omitted.

The TAR estimation results, summarized in Table 7, indicate that discrepancies between the results using, alternately, the interpolated and non-interpolated sectoral real exchange rates are fairly minor. In fact, for three of the five sectors the estimated delay parameter is identical for interpolated and non-interpolated sectoral real exchange rates (food, beverages and tobacco, textile and lumber and wood), while for the other two sectors estimated values for the delay parameter are relatively close ($d = 3$ and $d = 4$ for the manufacturing sector, while $d = 2$ and $d = 4$ for the paper sector). Estimates for the threshold parameter for the non-interpolated sectoral real exchange rates are generally consistent with estimated thresholds for the interpolated sectoral real exchange rates. For the paper and textile sector the nonlinear estimation produces identical estimates of the threshold parameter. For the remaining three sectors the difference in transaction and transportation costs between interpolated and non-interpolated sectoral real exchange rates is nonzero but negligible - 3% in the food, beverages and tobacco sector, 1% in the manufacturing sector and 2% in the lumber and wood sector.[27]

Next, the linearity test suggested by Hansen (1997) is applied to all sectoral real exchange rates. The bootstrap-calculated asymptotic p-values for the test of the null of linearity against the TAR alternative are presented in Table 7 and suggest that linearity is strongly rejected for all time series examined. Furthermore, p-values obtained in the case of the interpolated sectoral real exchange rates only differ slightly from the ones obtained from the non-interpolated sectoral real exchange rates. There is generally no significant evidence suggesting that interpolation leads to an over-rejection of the linear null hypothesis against the TAR alternative. The ratio of the residual variance of the estimated TAR model to the residual variance of an estimated linear AR(p) model, V, indicates for all estimated TAR models a significant reduction in the residual variance compared to the corresponding linear model. Overall, the ratio V shows a larger reduction for the sectoral real exchange rates generated using non-interpolated prices.

Table 8 reports the estimated roots and half lives for sectoral real exchange rates on the basis of annual data, non-interpolated quarterly data and interpolated quarterly data. In general the annual estimation results suggest longer half lives than the ones recorded for the quarterly data,

[27] The reported residual diagnostic statistics which include tests for serial correlation and autoregressive conditional heteroskedasticity do not provide any evidence of misspecifications in the residuals.

lending support to A.M. Taylor's (2001) result that temporal aggregation leads to upward bias in the convergence speed in this particular context. It is also evident from Table 8 that the interpolation of the price data yields estimated roots and half lives which are relatively close to the ones derived on the basis of the non-interpolated data.

To summarize, a comparison of the empirical findings for interpolated and non-interpolated sectoral real exchange rates suggests that, in this particular context, interpolation does not appear to cause invalid inferences.

6 Conclusion

The key objective of this study was to provide convincing evidence in favor of the presence of nonzero costs of international trade and in favor of the conjecture that deviations from the law of one price dissipate in a nonlinear, threshold-type fashion. Hence, we applied threshold autoregressive models to nine sectoral real exchange rates for five major industrialized countries over the recent floating period since 1974.

The nonlinear estimation results are encouraging. Estimated transactions costs appear to fluctuate widely across sectors and countries. It appears that goods markets between the US and Japan have lower transactions costs than between the US and Europe. These findings are generally consistent with the findings of Obstfeld and A.M. Taylor (1997). In general, adjustment towards the law of one price is observed to be fairly fast although the estimated delay parameter, which measures the timing of the reaction of market participants to deviations from the law of one price, is estimated to be longer than one might perhaps expect. In turn, this finding suggests that the delay parameter should be estimated instead of being set to unity *a priori*, as is often done in the relevant literature. Also, these results suggest that deviations from the law of one price may be somewhat sticky (given the delay parameter is on average larger than four quarters), but they are not as persistent as a large literature has hitherto suggested.

Our results also reinforce the argument of A.M. Taylor (2001) that there is a case for replacing the unit root null hypothesis with a stationary null hypothesis in the context of testing the validity of the law of one price since the process driving the deviations from the law of one price may well be stationary but have local unit roots in the inner regime. It would also be a great advance to produce data of the same quality used in this study at higher frequency in order to reduce the potential upward bias caused by temporal aggregation (A.M. Taylor, 2001).

Overall, this study provides strong evidence indicating the validity of the law of one price with allowance for the effects of transactions costs. In particular, our findings accord strongly with the emerging theoretical literature on nonlinear real exchange rate adjustment in the presence of international arbitrage costs and contribute towards forming a consensus view that discrete regime switching characterizes deviations from the law of one price across a broad range of goods for a number of industrialized countries over the recent floating period.

References

Andrews, D.W.K., Ploberger, W., 1994. Optimal Tests When a Nuisance Parameter Is Present Only under the Alternative. Econometrica 62, 1383-1414.

Balke, N.S., Fomby, T.B., 1997. Threshold Cointegration. International Economic Review 38, 627-45.

Benninga, S., Protopapadakis, A.A., 1988. The Equilibrium Pricing of Exchange Rates and Assets When Trade Takes Time. Journal of International Economics 7, 129-49.

Bergin, P.R., Feenstra, R.C., 2001. Pricing-to-Market, Staggered Contracts and Real Exchange Rate Persistence. Journal of International Economics 54, 333-59.

Betts, C., Devereux, M.B., 2000. Exchange Rate Dynamics in a Model of Pricing-to-Market. Journal of International Economics 50, 215-44.

Brock, W.A., Dechert, W.D., Scheinkman, J., 1991. Nonlinear Dynamics, Chaos, and Instability. MIT Press, Cambridge, MA.

Cassel, G., 1922. Money and Foreign Exchange After 1914. Constable, London.

Cheung, Y.-W., Chinn, M., Fujii, E., 2001. Market Structure and the Persistence of Sectoral Real Exchange Rates. International Journal of Finance and Economics 6, 95-114.

Cheung, Y.-W., Lai, K.S., 1993a. A Fractional Cointegration Analysis of Purchasing Power Parity. Journal of Business and Economic Statistics 11, 103-12.

Cheung, Y.-W., Lai , K.S., 1993b. Long-Run Purchasing Power Parity during the Recent Float. Journal of International Economics 34, 181-92.

Cheung, Y.-W., Lai, K.S., 2000a. On Cross-country Differences in the Persistence of Real Exchange Rates. Journal of International Economics 50, 375-97.

Cheung, Y.-W., Lai, K.S., 2000b. On the Purchasing Power Parity Puzzle. Journal of International Economics 52, 321-30.

Coakley, J., Fuertes, A.M., 1997. New Panel Unit Root Tests of PPP. Economics Letters 57, 17-22.

Coleman, A.M.G., 1995. Arbitrage, Storage and the Law of One Price: New Theory for the Time Series Analysis of an Old Problem. Discussion Paper, Department of Economics, Princeton University.

Davies, R.B., 1977. Hypothesis Testing when a Nuisance Parameter is Present Only Under the Alternative. Biometrika 64, 247-54.

Davies, R.B., 1987. Hypothesis Testing when a Nuisance Parameter is Present Only Under the Alternative. Biometrika 74, 33-43.

Davutyan, N., Pippenger, J., 1990. Testing Purchasing Power Parity: Some Evidence of the Effects of Transactions Costs. Econometric Reviews 9, 211-40.

Dixit, A.K., 1989. Hysteresis, Import Penetration and Exchange Rate Pass-Through. Quarterly Journal of Economics 104, 205-28.

Dumas, B., 1992. Dynamic Equilibrium and the Real Exchange Rate in Spatially Separated World. Review of Financial Studies 5, 153-80.

Edison, H.J., Gagnon, J.E., Melick, W.R., 1997. Understanding the Empirical Literature on Purchasing Power Parity: The Post-Bretton Woods Era. Journal of International Money and Finance 16, 1-17.

Engel, C., 1993. Real Exchange Rates and Relative Prices: An Empirical Investigation. Journal of Monetary Economics 32, 35-50.

Engel, C., Rogers, J.H., 1996. How Wide Is the Border? American Economic Review 86, 1112-25.

Feenstra, R.C., 1995. Exact Hedonic Price Indexes. Review of Economics and Statistics 77, 634-53.

Feenstra, R.C., Kendall, J.D., 1997. Pass-Through of Exchange Rates and Purchasing Power Parity. Journal of International Economics 43, 237-61.

Froot, K.A., Kim, M., Rogoff, K., 1995. The Law of One Price Over 700 Years. National Bureau of Economic Research, Working Paper No. 5132.

Froot, K.A., Rogoff, K., 1995. Perspectives on PPP and Long-run Real Exchange Rates. In Rogoff, K., Grossman, G. (Eds.). Handbook of International Economics. North Holland: Amsterdam.

Giovannini, A., 1988. Exchange Rates and Traded Goods Prices. Journal of International Economics 24, pp. 45-68.

Granger, C.W.J., Lee, T.-H., 1999. The Effect of Aggregation on Nonlinearity. Econometric Reviews 18, 259-66.

Granger, C.W.J., Teräsvirta, T., 1993. Modelling Nonlinear Economic Relationships. Oxford University Press: Oxford, UK.

Heckscher, E.F., 1916. Växelkursens Grundval vid Pappersmyntfot. Ekono-misk Tidskrift 18, 309-12.

Hansen, B.E., 1996. Inference when a Nuisance Parameter is not Identified under the Null Hypothesis. Econometrica 64, 413-30.

Hansen, B.E., 1997. Inference in TAR Models. Studies in Nonlinear Dynamics and Econometrics 2, 1-14.

Hendry, D.F., 1984. Monte Carlo Experimentation in Econometrics. In Griliches Z., M.D. Intrilligator (Eds.). Handbook of Econometrics. North-Holland: Amsterdam.

Isard, P., 1977. How Far Can We Push the "Law of One Price"? American Economic Review 67, 942-48.

Knetter, M.M., 1989. Price Discrimination by U.S. and German Exporters. American Economic Review 79, 198-210.

Knetter, M.M., 1993. International Comparisons of Price-to-Market Behaviour. American Economic Review 83, 473-86.

Knetter, M.M., 1994. Did the Strong Dollar Increase Competition in U.S. Product Markets? Review of Economics and Statistics 76, 192-95.

Krugman, P.R., 1987. Pricing to Market When the Exchange Rate Changes. In Arndt, S.W., J.D. Richardson (Eds.). Real-Financial Linkages Among Open Economies. MIT Press, Cambridge, MA, pp. 49-70.

Krugman, P.R., 1989. Exchange Rate Instability. MIT Press: Cambridge, MA.

Lothian, J.R., 1997. Multi-country Evidence on the Behavior of Purchasing Power Parity under the Current Float. Journal of International Money and Finance 16, 19-35.

Lothian, J.R., Taylor, M.P., 1996. Real Exchange Rate Behaviour: The Recent Float from the Perspective of the Past Two Centuries. Journal of Political Economy 104, 488-510.

Lothian, J.R.., Taylor, M.P., 2000. Purchasing Power Parity Over Two Centuries: Strengthening the Case for Real Exchange Rate Stability. Journal of International Money and Finance 19, 759-64.

Michael, P., Nobay, A.R., Peel, D.A., 1994. Purchasing Power Parity Yet Again: Evidence from Spatially Separated Markets. Journal of International Money and Finance 13, 637-57.

Michael, P., Nobay, A.R., Peel, D.A., 1997. Transactions Costs and Nonlinear Adjustment in Real Exchange Rates: An Empirical Investigation. Journal of Political Economy 105, 862-79.

Obstfeld, M., Rogoff, K., 2000. The Six Major Puzzles in International Macroeconomics: Is There a Common Cause? In Bernanke, B., Rogoff, K. (Eds.). National Bureau of Economic Research Macroeconomics Annual 2000. NBER and MIT Press: Cambridge, MA.

Obstfeld, M., Taylor, A.M., 1997. Nonlinear Aspects of Goods-Market Arbitrage and Adjustment: Heckscher's Commodity Points Revisited. Journal of the Japanese and International Economies 11, 441-79.

O'Connell, P.G.J., 1998. Market Frictions and Real Exchange Rates. Journal of International Money and Finance 17, 71-95.

O'Connell, P.G.J., Wei, S.-J., 2002. The Bigger They Are, The Harder They Fall: Retail Price Differences across U.S. Cities. Journal of International Economics 56, 21-53.

Officer, L.H., 1982. Purchasing Power Parity and Exchange Rates: Theory, Evidence and Relevance. JAI Press: Greenwich, CT.

Ohanian, L.E., Stockman, A.C., 1997. Arbitrage Costs and Exchange Rates. Discussion Paper, Department of Economics, University of Rochester, mimeo.

Parsley, D.C., Wei, S.-J., 1996. Convergence to the Law of One Price without Trade Barriers or Currency Fluctuations. Quarterly Journal of Economics 111, 1211-36.

Pippenger, M.K., Goering, G.E., 1993. A Note on the Empirical Power of Unit Root Tests under Threshold Processes. Oxford Bulletin of Economics and Statistics 55, 473-81.

Prakash, G., Taylor, A.M., 1997. Measuring Market Integration: A Model of Arbitrage with an Econometric Application to the Gold Standard, 1879-1913. National Bureau of Economic Research Working Paper No. 6073, June.

Ramsey, J.B., 1969. Tests for Specification Errors in Classical Least-Squares Regression Analysis. Journal of the Royal Statistical Analysis Series B, 31, 350-71.

Richardson, J.D., 1978. Some Empirical Evidence on Commodity Arbitrage and the Law of One Price. Journal of International Economics 8, 341-51.

Rogoff, R., 1996. The Purchasing Power Parity Puzzle. Journal of Economic Literature 34, 647-68.

Samuelson, P., 1954. The Transfer Problem and Transport Costs: Analysis of the Effects of Impediments. Economic Journal 65, 264-89.

Sarno, L., 2000a. Real Exchange Rate Behaviour in the Middle East: A Re-examination. Economics Letters 66, 127-36.

Sarno, L., 2000b. Systematic Sampling and Real Exchange Rates. Weltwirtschalftliches Archiv 136, 24-57.

Sarno, L., 2001. Toward a New Paradigm in Open Economy Modeling: Where Do We Stand? Federal Reserve Bank of St. Louis Review 83, 21-36.

Sarno, L., Taylor, M.P., 2002. Purchasing Power Parity and the Real Exchange Rate. International Monetary Fund Staff Papers, forthcoming.

Sercu, P., Uppal, R., Van Hulle, C., 1995. The Exchange Rate in the Presence of Transactions Costs: Implications for Tests of Purchasing Power Parity. Journal of Finance 50, 1309-19.

Taylor, A.M., 2000. A Century of Purchasing Power Parity. Review of Economics and Statistics, forthcoming.

Taylor, A.M., 2001. Potential Pitfalls for the Purchasing Power Parity Puzzle? Sampling and Specification Biases in Mean Reversion Tests of the Law of One Price. Econometrica 69, 473-98.

Taylor, M.P., Peel, D.A., Sarno, L., 2001. Nonlinear Mean-Reversion in Real Exchange Rates: Towards a Solution to the Purchasing Power Parity Puzzles. International Economic Review 42, 1015-42.

Taylor, M.P., Sarno, L., 1998. The Behaviour of Real Exchange Rates During the Post-Bretton Woods Period. Journal of International Economics 46, 281-312.

Tjostheim, D., 1986. Estimation in Nonlinear Time Series Models. Stochastic Processes and Their Applications 21, 251-73.

Tong, H., 1983. Threshold Models in Nonlinear Time Series Analysis. Springer-Verlag: New York, NY.

Tong, H., 1990. Nonlinear Time Series: A Dynamical System Approach. Clarendon Press: Oxford, UK.

Tsay, R.S., 1989. Testing and Modelling Threshold Autoregressive Processes. Journal of the American Statistical Association 84, 231-40.

Table 1. TAR estimation results: UK

	BMI	CHE	MEL	MEQ	FOD	MAN	PAP	TEX	WOD
θ	0.10	0.17	0.03	0.13	0.11	0.19	0.19	0.20	0.14
d	5	3	8	5	5	3	1	2	2
r	0.89	0.87	0.88	0.69	0.79	0.72	0.81	0.91	0.93
$Q(4)$	0.85	0.85	0.98	0.89	0.95	0.41	0.60	0.97	0.90
$Q(8)$	0.98	0.97	0.91	0.92	0.98	0.63	0.87	0.98	0.93
$ARCH(1)$	0.99	0.95	0.77	0.44	0.64	0.43	0.86	0.91	0.58
$ARCH(4)$	0.98	0.50	0.35	0.57	0.51	0.95	0.92	0.74	0.69
$ARCH(8)$	0.97	0.76	0.41	0.81	0.92	0.97	0.98	0.97	0.81
V	0.81	0.69	0.76	0.77	0.73	0.68	0.70	0.86	0.82
$DW - Stat$	1.94	1.95	2.08	1.99	1.91	2.01	2.01	2.03	1.94
$p - value$	0.05	0.002	0.05	0.06	0.005	0.000	0.007	0.16	0.06

Notes: Sectoral real exchange rates are estimated using a TAR(p,k,d) model, where in all cases the autoregressive order $p = 8$, the number of thresholds $k = 2$ (the thresholds being equal to $-\theta$ and θ respectively), and d denotes the delay parameter. The outer root of the TAR process, $r = \sum_{j=1}^{p} \beta_j$, i.e. it is defined as the sum of the estimated outer autoregressive parameters, and hence measures the degree of nonlinear mean reversion. Estimation by sequential least squares, as discussed in the text. $Q\,(4)$ and $Q\,(8)$ are Lagrange multiplier test statistics for up to fourth-order and eight-order serial correlation in the residuals respectively; $ARCH(i)$ is are Lagrange multiplier test statistics for autoregressive conditional heteroskedasticity in the residuals of order i. V is the ratio of the residual variance from each of the estimated TAR models to the residual variance from the corresponding linear autoregressive model. The p-value refers to the marginal significance level from executing the Hansen (1997) linearity test, constructed as discussed in the text.

Table 2. TAR estimation results: Germany

	BMI	CHE	MEL	MEQ	FOD	MAN	PAP	TEX	WOD
θ	0.07	0.17	0.07	0.08	0.20	0.17	0.01	0.19	0.18
d	5	5	8	3	5	4	5	1	8
r	0.91	0.93	0.91	0.94	0.79	0.79	0.90	0.77	0.81
$Q(4)$	0.90	0.86	0.99	0.97	0.99	0.82	0.76	0.99	0.99
$Q(8)$	0.98	0.96	0.98	0.96	0.99	0.96	0.97	0.99	0.99
$ARCH(1)$	0.30	0.93	0.52	0.75	0.26	0.61	0.67	0.91	0.45
$ARCH(4)$	0.67	0.93	0.68	0.42	0.72	0.90	0.92	0.87	0.63
$ARCH(8)$	0.35	0.23	0.42	0.73	0.70	0.76	0.90	0.89	0.67
V	0.65	0.82	0.81	0.82	0.77	0.81	0.77	0.81	0.81
$DW-Stat$	1.76	1.88	2.03	2.00	1.88	1.92	1.94	2.03	1.97
$p-value$	0.001	0.05	0.05	0.12	0.03	0.02	0.02	0.02	0.05

Notes: See Notes to Table 1.

Table 3. TAR estimation results: France

	BMI	CHE	MEL	MEQ	FOD	MAN	PAP	TEX	WOD
θ	0.04	0.19	0.17	0.18	0.13	0.19	0.05	0.14	0.12
d	5	2	8	8	8	4	5	1	4
r	0.94	0.82	0.88	0.86	0.39	0.74	0.89	0.93	0.66
$Q(4)$	0.91	0.96	0.93	0.99	0.54	0.81	0.76	0.99	0.89
$Q(8)$	0.98	0.99	0.99	0.95	0.83	0.98	0.72	0.99	0.90
$ARCH(1)$	0.35	0.76	0.96	0.99	0.59	0.61	0.55	0.20	0.68
$ARCH(4)$	0.23	0.93	0.81	0.79	0.97	0.97	0.68	0.48	0.98
$ARCH(8)$	0.49	0.78	0.93	0.92	0.53	0.96	0.81	0.89	0.94
V	0.75	0.86	0.81	0.80	0.73	0.75	0.71	0.86	0.75
$DW-Stat$	1.88	1.87	1.97	2.00	2.08	1.85	1.88	1.91	1.93
$p-value$	0.01	0.12	0.08	0.03	0.08	0.11	0.01	0.09	0.005

Notes: See Notes to Table 1.

Table 4. TAR estimation results: Italy

	BMI	CHE	MEL	MEQ	FOD	MAN	PAP	TEX	WOD
θ	0.04	0.02	0.18	0.20	0.20	0.20	0.16	0.19	0.20
d	5	4	8	6	1	7	1	4	4
r	0.94	0.94	0.74	0.52	0.94	0.19	0.79	0.93	0.61
$Q(4)$	0.68	0.93	0.74	0.90	0.89	0.61	0.91	0.99	0.89
$Q(8)$	0.94	0.98	0.75	0.96	0.98	0.93	0.98	0.99	0.98
$ARCH(1)$	0.03	0.67	0.50	0.56	0.66	0.80	0.72	0.93	0.58
$ARCH(4)$	0.08	0.57	0.77	0.68	0.73	0.68	0.91	0.87	0.35
$ARCH(8)$	0.10	0.73	0.61	0.37	0.06	0.74	0.98	0.19	0.29
V	0.60	0.87	0.77	0.74	0.73	0.74	0.66	0.83	0.68
$DW-Stat$	1.97	1.96	1.97	1.86	1.85	1.81	1.80	1.94	1.91
$p-value$	0.000	0.02	0.02	0.05	0.001	0.008	0.000	0.07	0.001

Notes: See Notes to Table 1.

Table 5. TAR estimation results: Japan

	BMI	CHE	MEQ	FOD	MAN	PAP	TEX
θ	0.04	0.05	0.17	0.07	0.14	0.05	0.06
d	5	5	4	8	5	7	1
r	0.94	0.96	0.92	0.94	0.85	0.87	0.95
$Q(4)$	0.99	0.89	0.89	0.96	0.92	0.98	0.99
$Q(8)$	0.98	0.86	0.97	0.97	0.96	0.97	0.99
$ARCH(1)$	0.54	0.54	0.76	0.26	0.43	0.44	0.92
$ARCH(4)$	0.97	0.04	0.53	0.61	0.29	0.71	0.85
$ARCH(8)$	0.96	0.16	0.33	0.60	0.17	0.93	0.75
V	0.79	0.78	0.89	0.75	0.74	0.80	0.77
$DW-Stat$	2.03	2.02	2.05	1.93	2.01	2.02	1.88
$p-value$	0.04	0.01	0.02	0.01	0.02	0.01	0.01

Notes: See Notes to Table 1.

Table 6. Power of Hansen Linearity Test

	$r = 0.95$ $TE = 0.05$	$r = 0.75$ $TE = 0.25$	$r = 0.5$ $TE = 0.5$	$r = 0.25$ $TE = 0.75$	$r = 0.05$ $TE = 0.95$
$p = 8$, $T = 80$	55.0	61.0	68.4	77.0	85.0
$p = 4$, $T = 80$	60.0	66.0	75.0	85.0	92.0
$p = 2$, $T = 80$	69.0	74.8	87.0	94.0	98.2

Notes: In all cases rejection frequencies are at a nominal significance level of five percent and are calculated on the basis of $1,000$ replications. The number of lags used in the data generating process is p, T is the number of data points, r measures the degree of mean reversion, and TE denotes the threshold effect. The data generating process is a TAR process calibrated using the estimation results from the TAR estimation of the real exchange rate data for the UK paper sector. Precise details of the Monte Carlo experiments are given in the text.

Table 7. TAR Estimation Results: Non-Interpolated and Interpolated Sectoral Real Exchange Rates

	q_{NI}					q_I				
	RFOD	RMAN	RPAP	RTEX	RWOD	RFOD	RMAN	RPAP	RTEX	RWOD
θ	10	3	1	12	12	13	2	1	12	14
d	1	3	2	3	2	1	4	4	3	2
$Q(4)$	0.97	0.90	0.96	0.93	0.93	0.98	0.95	0.91	0.95	0.91
$Q(8)$	0.99	0.88	0.96	0.75	0.82	0.99	0.96	0.98	0.98	0.99
$ARCH(1)$	0.24	0.26	0.72	0.49	0.11	0.95	0.70	0.93	0.57	0.74
$ARCH(4)$	0.51	0.70	0.63	0.55	0.23	0.94	0.78	0.64	0.86	0.78
$ARCH(8)$	0.64	0.78	0.34	0.22	0.20	0.92	0.91	0.08	0.21	0.87
V	0.83	0.88	0.74	0.76	0.70	0.84	0.81	0.81	0.83	0.78
$DW-Stat$	1.96	2.02	2.15	2.05	2.07	1.98	2.09	2.09	1.94	2.09
$p-value$	0.02	0.004	0.002	0.00	0.01	0.00	0.02	0.02	0.00	0.01

Notes: q_{NI} (q_I) denotes the sectoral real exchange rate constructed using non-interpolated (interpolated) price data. Sectoral real exchange rates are estimated using a TAR(p,k,d) model, where the autoregressive order $p = 8$, the number of thresholds $k = 2$ (the thresholds being equal to $-\theta$ and θ respectively), and d denotes the delay parameter. $Q(4)$ and $Q(8)$ are Lagrange multiplier test statistics for up to fourth-order and eight-order serial correlation in the residuals; $ARCH(i)$ is a Lagrange multiplier test statistic for autoregressive conditional heteroskedasticity in the residuals of order i. V is the ratio of the residual variance from each of the estimated TAR models to the residual variance from the corresponding linear autoregressive model. *DW-Stat* denotes the Durbin-Watson test statistic. The p-value refers to the marginal significance level from executing the linearity test (Hansen, 1997).

Table 8. Estimated Root and Half Lives

	RFOD	RMAN	RPAP	RTEX	RWOD
Annual					
r	0.51	0.42	0.31	0.68	0.12
hl	4.1	3.2	2.4	7.2	1.3
$\text{Quarterly}_{\text{NI}}$					
r	0.44	0.81	0.88	0.76	0.34
hl	0.8	3.3	5.4	2.5	0.6
$\text{Quarterly}_{\text{I}}$					
r	0.48	0.75	0.84	0.80	0.31
hl	0.9	2.4	4.0	3.1	0.6

Notes: The table displays the estimated root (r) and half life (hl) obtained from fitting a TAR model to five sectoral real exchange rates for the UK. Estimated roots and half lives are computed using annual and quarterly data. Data at quarterly frequency are distinguished between non-interpolated data ($\text{Quarterly}_{\text{NI}}$) and interpolated data ($\text{Quarterly}_{\text{I}}$). All half lives are reported in quarters.